

# **Industrial Fiber Optics**

## **PHYSICAL OPTICS USING A HELIUM-NEON LASER**

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**IFO 45-788**

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## SAFETY NOTES

Lasers are valuable sources of light for exciting demonstrations and laboratory experiments in schools.

Industrial Fiber Optics helium-neon lasers emit a beam of visible orange-red light. Invisible, exotic, or otherwise harmful radiation is not emitted.

Industrial Fiber Optics lasers are low power lasers. With a light output of only a few thousandths of a watt, these lasers should not be confused with the powerful lasers intended for burning, cutting, and drilling.

Even though the power of IFO lasers is low, the beam should be treated with caution and common sense because it is intense and concentrated. The greatest potential for harm with IFO lasers is to the eyes. No one should look directly into the laser beam or stare at its bright reflections, just as no one should stare at the sun or arc lamps.

The United States Department of Health, Education, and Welfare regulate the manufacturers of lasers to see that users are not endangered. The federal government classifies lasers according to their power levels and specifies appropriate safety features for each level. Demonstration lasers fall into Class II and can be identified by a yellow **“CAUTION”** label that contains the warning **“Do not stare into beam”** as well as the universal laser warning symbol. Class II lasers have a maximum power of **1/1000<sup>th</sup>** of a watt, a power judged to be eye-safe, except possibly in case of deliberate, long-term direct staring into the beam. Safety features include a pilot lamp that glows when the electrical power is “ON” and a mechanical beam stop that blocks the beam when the power is on. In addition safety directives issued by the European Standards Committee for optical, electrical and other safety have been followed, in particular directives 89/336/EEC and 92/59/EEC for lasers and EN 71-1 for optics kits.

### Laser Safety

1. Instruct students not to look into the laser or stare at bright mirror-like reflections of the beam.
2. Block the beam beyond the farthest point of interest. Use a dull, non-reflective object, like a piece of wood.
3. If the beam must travel a long distance keep it close to the ground or overhead so that it does not cross walkways at eye level.
4. Never let unauthorized people handle a laser; store the laser in a safe place away from unauthorized people.
5. Lasers are not toys; use them only for educational purposes.
6. Never point a laser at anyone.
7. Make sure the laser (and its transformer) is always secured on a solid foundation.
8. Helium-neon lasers employ high internal voltages; the power supply retains the potentially harmful voltage for periods after the input power has ceased. Never open the housing and expose yourself to these voltages.
9. Keep these safety regulations near the laser, read and refer to them in case of safety questions.
10. If you have any other safety questions, please contact Industrial Fiber Optics.

### General Safety

1. Since the optics kits contain glass pieces, it is important to remember that they can pose a cutting hazard. Please handle them carefully and make sure that you don't contact any edges that might cut your skin.
2. In order to fully utilize this manual and its safety warnings, it is recommended that the manual be kept in close proximity to the optics lab.

### Electrical Safety

If the laser housing is opened, Industrial Fiber Optics warranty will be voided.

Each laser is equipped with a UL approved line cord and three prong grounded plug. **ALWAYS PLUG THE LASER INTO A GROUNDED OUTLET.**

For further information on Laser Safety and the Federal Regulations involved, you are advised to contact the Compliance Officer at the Bureau of Radiological Health, Public Health Service, Food & Drug Admin., Rockville, MD 20857, Tel No. 301/443-4874. Ask for Regulation Publication HHS PUB FDA 80-80356.

## PREFACE

The structure of this manual and its content is based on a previous edition written by Arthur Eisenkraft for Metrologic Inc. While the content has been rewritten for Industrial Fiber Optics, the sequence and form of the experiments has been maintained to provide continuity for teachers and instructors already using Metrologic's Physical Optics Set (now provided by Industrial Fiber Optics) in their curriculums.

## INTRODUCTION

Students are often exposed to important mathematical concepts that can be difficult to understand because they're not intuitive or presented using practical applications. Fourier Transformation is a mathematical topic frequently taught in advanced courses of study. Explanations of this important concept are usually supplemented with graphical aids such as waveforms and plots showing distribution of energy vs. frequency, etc. These can help students grasp what can be done with Fourier Transforms from a mathematical point of view in what could be considered a virtual presentation form. But most people relate to and become more excited about real world results and that is where laboratory studies can be beneficial. At their best, laboratory studies show how scientific concepts are applied, and can inspire students to appreciate why they are important in everyday life.

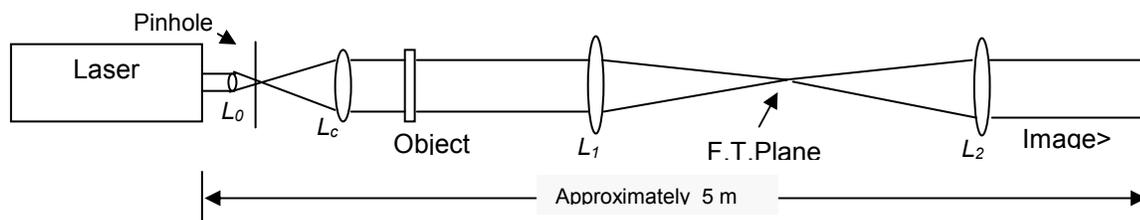
This manual presents a series of optical laboratory activities typically not covered by introductory physics courses in high schools and universities. The activities are structured to develop an experimental setup for demonstrating Fourier Transformation. It's assumed the student has an understanding of basic physics, laser behavior, the concept of interference, and the effects caused by lenses. An important aspect of these experiments is that the student gains experience in the practical details of working with a laser and setting up lenses to achieve an experimental result. But the ultimate goal is to demonstrate Fourier Transformation in a way that bridges the gap between mathematical theory and real world application.

The material in this manual is written for use with Industrial Fiber Optics Physical Optics Lab 45-688 and the ML-8xx series of HeNe lasers. However, the information and experimental setups can be used generically if equivalent optical components are substituted. The manual contains text, illustrations and photographs that clearly outline the procedures needed for successful results. But an important value in the lab activities is that students gain a deeper understanding of how the various elements affect the results. Unlike a static diagram on paper, the lab experiments are performed "real time" and by nature are interactive.

The optical lens is a key element used in the experiments within this manual. Ordinarily it's the refractive properties of a lens, which give it the ability to project, or focus images that's of primary interest. In this manual students will experience a different aspect of lens operation - the ability to perform a Fourier Transform operation on a two dimensional image. The general term for using optical components in this way is called Fourier Optics. To appreciate the importance of this branch of optics it helps to understand what Fourier Transformation is and how it's generally done.

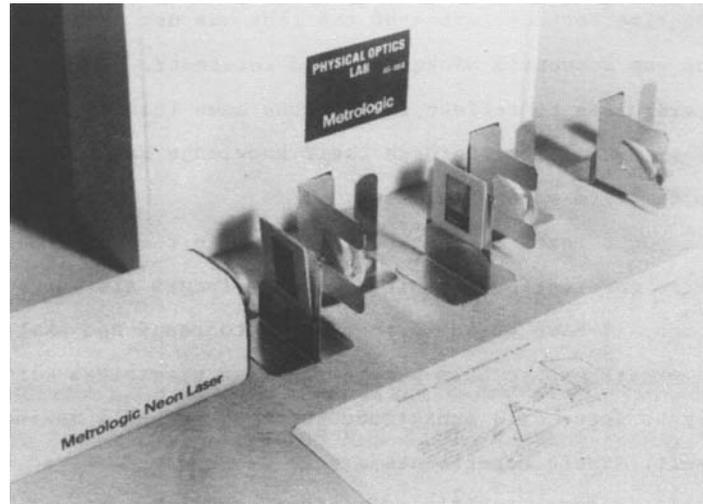
Fourier Transformation is ordinarily used in signal and information processing to determine the frequency content of a signal source. If you wonder what practical use this might have, consider an application like speech recognition. Each person has a unique distribution of energy at various frequencies in the audio spectrum, much like a fingerprint. Recognition generally requires decomposing the complex waves associated with speech to sort out these components. Fourier Analysis is a mathematical method used to take a complex signal, often in electrical form, to determine these frequency components. Usually this is achieved by digitizing the signal to generate data, after which computer algorithms are used to perform the Fourier Transformation. In principle this might seem easy to do using modern computing power, but in practice the computations can be intensive. Pattern recognition in images is another application for Fourier Transformation, and the same digitizing and computational issues apply.

But it happens that under certain conditions a lens can perform a Fourier Transformation directly. The result of this transformation can be analyzed with far less computational power than would be needed if the computer was digitizing and applying algorithms to the original source content. This lab manual will demonstrate the principles of this method by establishing the experimental setup shown below:



A laser provides a source of parallel coherent light. The laser beam is focused, "filtered" with a pinhole to remove noise, and then expanded with a lens to a size suitable for illuminating an object. Exposed to monochromatic coherent light, the object generates a diffraction pattern. Another lens brings this pattern into focus on a screen placed at the F.T. (Fourier Transform) plane. This is where the decomposed frequency, intensity and phase content of the object image are available for viewing or manipulation. The last lens in the setup performs an inverse Fourier Transform to recreate a representation of the original object image. It's important to note that the manipulation possible at the F.T. plane is a powerful aspect of this process. For instance, by blocking some of the patterns appearing at the F.T. plane, a "filtered" version of the image can be generated. This would be similar to listening to audio after manipulation by

an equalizer, or viewing a photograph or digital image after applying filtering techniques to adjust brightness, contrast, color, sharpness etc.



Photograph of setup

The activities after this introduction follow a sequence that builds familiarity with the individual elements of the experimental setup. As each element is introduced its effects on the development of the Fourier Transformation will be seen and manipulated. Setting up and aligning the optical components provides an interaction with the transformation process that improves understanding of the observed effects. Once the setup is established subsequent activities will show how Fourier Transformation, often regarded as a mathematical tool, can be applied in dramatic ways to practical tasks in the physical world.

# ACTIVITY 1: SPATIAL FILTERING AND COLLIMATION

## Overview

This activity will concentrate on setting up a clean optical “engine” to provide a source for illuminating the object that will undergo a Fourier Transformation. A laser is the primary optical energy source, but as you’ll discover they often have noise components that could degrade the results. This will be observed and filtered, and then the beam will be expanded to a useful size with an appropriate lens.

## ACTIVITY 1.1 - OBSERVING OPTICAL NOISE

### Background and Procedure

Noise is present in all man made sources of energy. How it’s manifested or perceived depends on what method is used to intercept the energy. Our ears hear noise as “hiss” or perhaps distortion in what should otherwise be a clean audio recording. On a television set we might see “snow” or a grainy picture if a weak “over the air” signal is intercepted by an antenna. On a cable connection with a strong signal we might still see noise as a visual distortion in a digitally manipulated image. Noise can be defined as any unintended content that adds no value or degrades a desired source of energy, signal, or information.

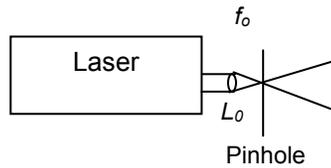
In the optical setup you’ll use for demonstrating Fourier Transforms there are several sources of noise. The laser tube generates noise, often due to imperfection in the mirrors, bore, operational stability, dust particles captured during manufacture, etc. The noise shows up as stray light or undesired patterns around the main beam. Lenses in the setup are another source of optical noise. Dust and imperfections within the lens can cause unwanted interference patterns that have nothing to do with the object being tested. Basically, anywhere a coherent optical beam is generated, passes through an interface, or intercepts a target, is a potential source of noise.

Assuming the lenses are clean and of good quality, the main source of optical noise will be the laser. You can see the noise by shining the laser beam on a flat white reflective surface 1 or 2 meters away. Stray light and non-uniform patterns will likely appear around what should be a clean spot of laser light. Fortunately there’s a fairly simple technique that will eliminate most of this noise. It involves using a spatial filter, which is essentially a pinhole through which the laser beam is focused to strip off many of these unwanted artifacts. The following activity will describe using the spatial filter contained in the Industrial Fiber Optics Physical Optics Set, or as an alternate, constructing one from simple materials.

## ACTIVITY 1.2 - MAKING A SPATIAL FILTER

### Overview

A spatial filter works by exploiting the fact that stray or off axis light from a laser will focus at a different point in space than the desired main beam. The main components of a spatial filter are a focusing lens and pinhole as shown in the following figure:



The lens focuses the laser beam to a pinpoint, and a pinhole is placed at the focal point of the lens. Optical energy from noise generated in the laser focuses at different locations than the main beam – basically in a larger concentric circle around the main beam at the focal point of the lens. By making the pinhole large enough to just pass the main beam the remaining optical noise energy is blocked. A spatially filtered laser beam generates a clean spot with uniform illumination when viewed on a flat white reflecting screen. Rings or other artifacts around the spot would indicate a pinhole that was too large or a misaligned setup.

### Procedure

Industrial Fiber Optics Physical Optics Lab contains a matched lens and pinhole set. The lens is 9 mm in diameter with a +15 mm focal length, and is mounted in a  $\frac{3}{4}$ "-32 TPI (threads per inch) threaded carrier. This matches the threads on the optics mounts of Industrial Fiber Optics lasers, permitting direct insertion. The +15 mm lens focuses the beam to approximately a 25 micron (um) diameter. The pinhole selected for this lens has a 50 um diameter aperture.

Any short focal length positive lens may be substituted for this lens. A 10x-microscope objective works well, as long as the edge of the beam does not strike the side of the lens.

A substitute for the pinhole can be made by placing 8 to 10 layers of aluminum foil on a piece of glass. Puncture the aluminum using a sewing needle or pin, and then select a hole of appropriate size by trial and error. A good pinhole will usually be found in the middle layers.

The spatial filter is the combination of lens and pinhole. In order for it to work properly it's crucial that these two elements are properly aligned. This is done in two steps – centering the focusing lens on the laser beam, then placing and centering the pinhole at the focal point of the lens. Proceed as follows for the Industrial Fiber Optics +15 mm lens # 45-629:

1. Place a piece of white paper or other viewing surface about 0.5 meters in front of the laser.
2. Turn on the laser and mark the beam position on the screen with a pencil.
3. Keep the laser steady and screw the +15 mm lens into the threads of the laser optics mount.
4. The image on the screen will be an expanded spot. Determine if it's centered around the pencil mark on the screen.
5. If the spot is off-center, use a small hex wrench to loosen the three screws that hold the optics mount in place. Do not move the laser. Push the optics mount until the spot is centered on the pencil mark and tighten the screws.

The focal point of the laser beam is approximately 15 mm from the +15 mm lens. Dim the room lights to help monitor the alignment on the viewing screen. Position and center the pinhole # 45-674 at the laser beam focal point as follows:

1. Place a viewing screen about 0.3 meters beyond the lens.
2. The pinhole is small, and getting it to an initial position where the laser beam passes through the pinhole will require trial and error. Use the projected spot image on the viewing screen as a guide. When you get the first indications that the laser beam is passing through the pinhole, make any additional movements small and precise. Use the following indications to help your alignment:
  - a. When the pinhole is centered within the beam, but too close or too far away from the lens, a set of concentric rings will appear on the screen.
  - b. When the correct distance is reached, a uniformly illuminated spot of light will appear on the screen. The spot will be about as bright as if no pinhole has been used.

### **ACTIVITY 1.3 - BEAM EXPANSION AND COLLIMATION**

#### **Overview**

In many of the exercises in Activities 2 through 6 an object will be exposed to the laser beam to develop a diffraction pattern. To get best results it's important that the beam is highly collimated. A collimated beam behaves as if the light source was infinitely small (point source) and at an infinite distance from the observer. The nature of the light rays reaching the observer is that they are completely parallel, exhibiting no spread or divergence with distance. A star on a clear night is a reasonable example of this condition. The distance between most stars and the Earth is large enough that they simulate a point source. Starlight reaching the earth has highly parallel rays – you can move many miles while observing a star yet it will seem like it's in the same part of the sky.

The beam coming out of the spatial filter is rapidly diverging which can be observed by looking at how the spot size increases at several distances. A collimated beam between 20 and 30 mm diameter is needed to fully illuminate the photographic transparencies and other objects used in the exercises. We can develop this beam by placing a large diameter converging lens with a suitable focal length after the spatial filter.

Collimating lens # 45-664 in the Physical Optics Set has a focal length of 350 mm. When combined with the spatial filter it provides a beam that stays 25 mm in diameter over a distance of 10 meters.

#### **Lens alignment and Autocollimation**

There are several options for aligning the collimating lens used with the spatial filter. One method is to place a reflective target at a significant distance (ideally it would be at infinity...), and then adjust the lens position relative to the spatial filter until the projected spot size is at its smallest. This places the pinhole exactly at the focal point of the lens, which ensures that rays from the laser beam are parallel. The converse is also true – parallel rays entering the lens are brought exactly into focus at the focal point of

the lens. The technique can be approximated fairly well if the laboratory space has several meters of working distance and isn't too brightly lit. But this is often impractical and in that case another method known as autocollimation can be used.

Autocollimation exploits the reciprocity of lens operation. By using a mirror as a target for the collimating lens, the laser beam is reflected back to the source, which in the case of the spatial filter is the pinhole. The result is that a reflected pinhole image appears at the pinhole plane if the mirror is positioned perpendicular to the collimating lens. The autocollimation alignment then involves adjusting the collimation lens distance relative to the pinhole until the reflected image is centered over the pinhole and at it's smallest. The setup for autocollimation is shown below:

### **Procedure**

1. Position collimating lens # 45-664 ( $L_c$ ) about one focal length away from the spatial filter pinhole. If you have a nearby wall or target roughly establish this distance by moving the lens back and forth until the projected spot is at a minimum. To save time in the following steps try to get the beam lined up (bore sighted) with the laser tube axis.
2. Place mirror # 45-647 10 to 20 cm beyond lens  $L_c$  and tilt it so the laser beam is reflected back and centered on the lens surface.
3. You should see a bright image spot of the pinhole focused somewhere near the pinhole itself. Adjust lens  $L_c$  and tilt the mirror until the image spot is the smallest possible size and close to, or ideally, overlapping the pinhole. This will take some trial and error so be patient and methodical.
4. Remove the mirror and check the collimation. If the lens has been positioned correctly, the beam will remain the same diameter for a long distance. Check this by aiming the beam at a white reflective target and observe the beam size at various distances from the laser.

## ACTIVITY 2: GENERATING AND OBSERVING DIFFRACTION PATTERNS

### Overview

In this activity you'll generate diffraction patterns from various objects and discover how they appear different depending on whether they're formed near the object or at a considerable distance. You'll also observe the diffraction pattern from a narrow slit and see how the width affects the pattern spacing. Finally, you'll intercept the diffraction pattern with a lens at the correct location to perform a Fourier Transformation.

### ACTIVITY 2.1 - GENERATING AND VIEWING DIFFRACTION PATTERNS WITH SOLID OBJECTS

#### Background:

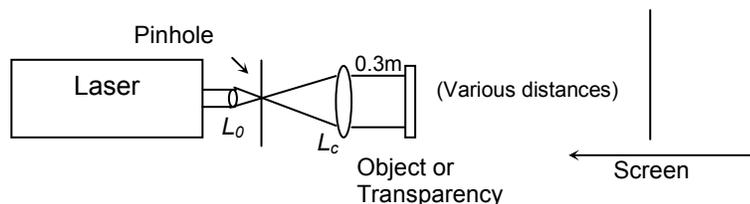
Diffraction patterns are something we can observe that confirms the wave behavior of light. We don't witness light's wave behavior often in daily circumstances unless we're treated to a rainbow, or see the visual effect of a thin oil film on water. But artificial conditions can be setup that dramatically demonstrates the wave nature of light, as you'll see in this activity.

Diffraction patterns result from constructive and destructive interference when a solid object blocks coherent, collimated, monochromatic light. The edge of the solid object generates a new wave front that eventually combines with the original unobstructed light wave. Since the wave fronts travel different path lengths they can add or subtract at various points away from the object edge, much like two ripples converging in a pond. From a practical point of view for the remaining exercises, the laser and spatial filter setup in Activity 1 meets the criteria of "coherent, collimated, and monochromatic".

Diffraction patterns observed at a distance close to the object have a different visual appearance than those viewed at large distances. The close or finite case is termed Fresnel or near-field diffraction. Fraunhofer or far-field diffraction is the term for patterns viewed at large distances or infinity.

#### Procedure

Activities 2.2, 2.3, and 2.4 use the laser and spatial filter setup developed and aligned in Activity 1. You'll place solid objects and transparencies inside the collimated laser beam and view the diffraction patterns on a viewing screen as shown below:



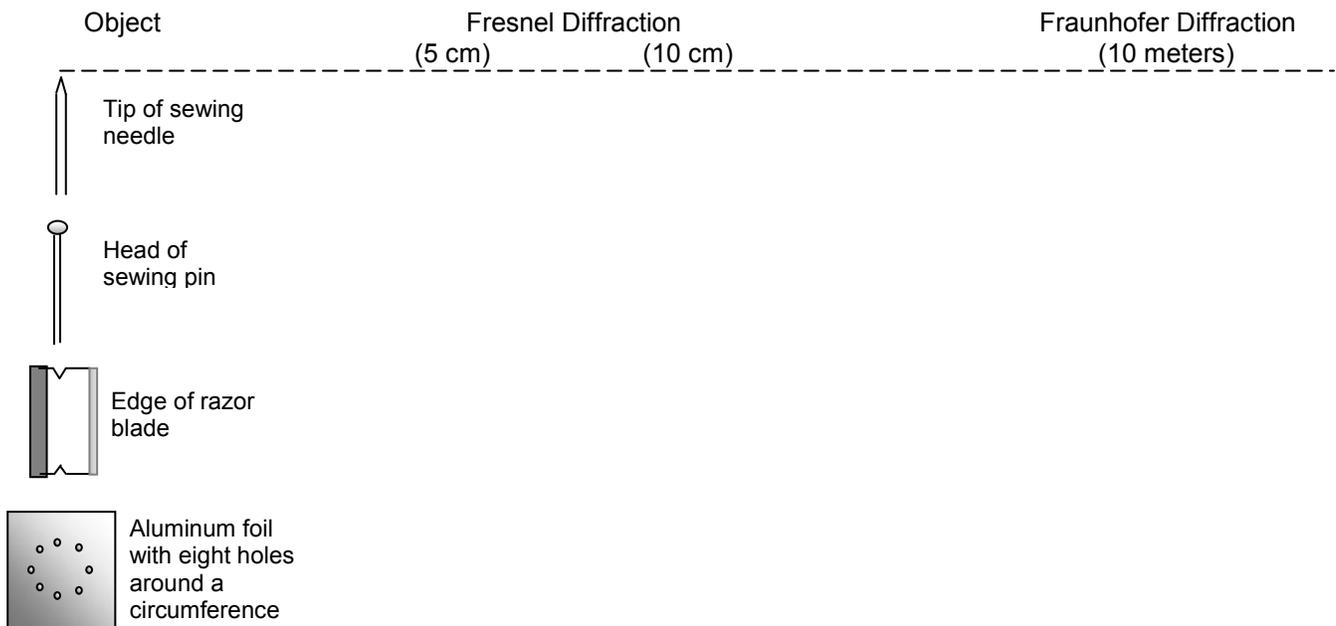
Viewing screen # 45-671 included in Industrial Fiber Optics Physical Optics Set is a matte Mylar plastic mounted in a photographic transparency holder. The collimated laser beam is expanded sufficiently to eliminate eye safety concerns when using low power Class II lasers for the activities in this manual. You can view the diffraction patterns by looking through the screen at the collimated laser beam. A good alternate viewing surface is ground glass. Regardless of which viewing surface is used, you can position either one at a slight angle to make viewing the patterns more comfortable.

A small optical magnifier will let you see more detail in the diffraction pattern. While a magnifying glass used for books and newspapers might be helpful, to get the best results use an eye loupe with roughly 4X to 10X magnification.

Gather the following common objects and place each one inside the collimated beam roughly 0.3 meters from lens  $L_c$ . The exact distance from the lens to the object is not critical.

1. Tip of sewing needle
2. Head of sewing pin
3. Edge of razor blade
4. Aluminum foil with eight evenly spaced pinholes in a 5 mm-diameter circle. Form the holes by using a needle or pin to pierce several layers of foil on a piece of glass. Use the foil from one of the middle layers.

View and sketch the diffraction patterns from each object at close or “finite” distances (5 cm and 10 cm), and at far or “infinite” distance (10 meters or more).



## ACTIVITY 2.2 OBSERVING HOW SLIT WIDTH AFFECTS THE DIFFRACTION IMAGE.

### Background

In Activity 2.1 you viewed the diffraction pattern from a single razor blade. By placing a second razor blade opposite the first you'll produce a second diffraction pattern. As you bring the two razors together to form a slit, the patterns will merge. Adjusting the slit width will change the spacing and brightness of the interference fringes. This relationship between the physical proximity of objects and observed interference patterns is often used in laboratory and industrial settings for accurate mechanical measurements. For instance, optical flats are used for precision surface inspection by viewing the interference fringes between the flat and a work piece when illuminated by a uniform monochromatic source. The air gap between the two when they are pressed together is small, but variations in the gap because of surface irregularities produce changes in the resulting interference pattern. The measuring precision for this type of inspection is within fractions of the light source wavelength. Further information on measurements by diffraction can be found in Industrial Fiber Optics "Experiments Using a Helium Neon Laser".

### Procedure

Use the setup from Activity 2.1 to determine how the spacing of the interference fringes in the diffraction pattern changes with physical separation between two objects. Place two razor blades in the beam so they are in the same plane and are parallel to each other. Initially space the blades to form a slit approximately 1 mm wide. Position the viewing screen roughly 10 cm from the razor blades, then adjust the gap between the blades and observe how the interference fringe spacing changes.

Describe how the razor blade separation affects the spacing of the interference fringes.

.....  
.....

## ACTIVITY 2.3 VIEWING THE DIFFRACTION PATTERNS FROM PHOTOGRAPHIC TRANSPARENCIES AT FINITE DISTANCES AND DISTANCES WHICH APPROXIMATE INFINITY.

### Overview

In this activity you'll view the diffraction patterns from photographic transparencies. They are a convenient means of demonstrating the effects that various physical configurations have on the coherent collimated beam. The transparencies save significant time that would otherwise be needed to fabricate and set up those configurations.

### Procedure

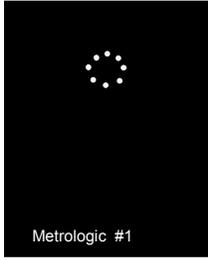
Place the following transparencies from Industrial Fiber Optics # 45-673 in the setup from Activity 2.1, then view and sketch the Fresnel and Fraunhofer diffraction patterns at the indicated distances.

**Objects with resulting Fresnel and Fraunhofer Diffraction patterns**

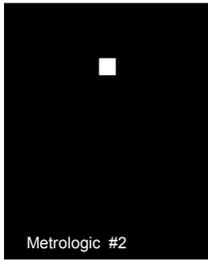
(5 cm)

(10 cm)

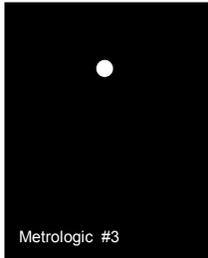
(10 meters)



#1 Eight holes in a circle



#2 Square aperture



#3 Circular aperture

**Problems:**

Describe any differences in shape between the Fresnel and the Fraunhofer patterns. ....

.....  
.....  
.....

Describe any differences in sharpness between the Fresnel and the Fraunhofer patterns. ....

.....  
.....  
.....

## ACTIVITY 2.4 VIEWING THE OPTICAL FOURIER TRANSFORM OF TRANSPARENCIES.

### Background

A complex result is usually the sum of many individual components. Think about a symphony orchestra performing a classical composition, or a Rock Band playing a popular tune. In each case what you hear is the sum of many unique components - instruments, musicians, vocal artists etc. The same can be said regarding observations of natural phenomena, which are often the result of several effects. A rainbow is caused by the sun emitting radiation traveling through space and entering our atmosphere, and then passing through falling rain or mist. What's notable in all these examples is that the results are produced by the sum of *unique* components.

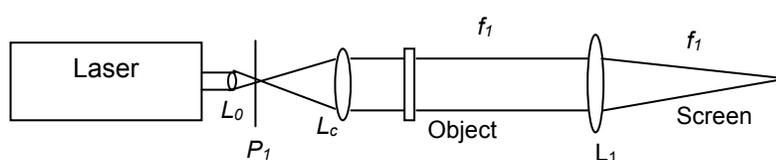
Wave motion is a fundamental mechanism for propagating energy, whether it's in the form of signals, light, sound etc. We've all experienced wave motion ranging from simple (ripples on water), to complex (the examples in the first paragraph) The French physicist Fourier proved that complex waves could be broken down into a number of simple waves. His method to mathematically decompose repetitive complex waves into simple ones is known as Fourier Transformation. An important aspect of Fourier Transformation is *that no matter how complex the waveshape appears, it can always be broken down into components of various frequencies derived from one particular waveshape - the sinusoid*. This is a powerful mathematical tool, because it means that a well understood, fundamental method of wave propagation can be used to synthesize an infinite variety of unique waveshapes.

A lens can perform a Fourier Transformation without using any difficult mathematics in the process. The transformation result may still require further analysis, but the difficult mathematical component is eliminated. When collimated coherent light is diffracted and passed through a lens, the lens optically performs the transform function. The result is the formation of a diffraction image, the Fourier Transform, which provides amplitude, phase, and position information of the original waveform.

In this exercise you'll place a lens along the path where Fresnel diffraction is ordinarily observed. The object will be located one focal length in front of the lens, and the Fourier Transform will be developed one focal length beyond the lens. The diffraction pattern formed after the transformation will appear similar to the far field or Fraunhofer pattern.

### Procedure

To perform a Fourier Transformation we'll add a lens to the setup as shown below:



1. Position transparency #1 (Eight holes in a circle) inside the collimated beam roughly 0.3 meters from collimating lens ( $L_c$ ) - the distance isn't critical.

2. Place long focal length lens Industrial Fiber Optics # 45-665 ( $L_1$ ) one focal length away from the transparency. Substitute long focal length lenses can be used, the longer the focal length the more spread out the diffraction image. A lens roughly 50 mm in diameter with a 1 meter focal length is recommended. A good substitute is a +1 diopter eyeglass lens used for reading as found in Pharmacies or Department stores. You'll use the autocollimation technique from Activity 1.3 to position the lens one focal length from the object, but without the laser - it's coherent properties causes diffraction making autocollimation difficult. Instead, temporarily place a point source of light (like a small non-frosted incandescent lamp) between collimating lens  $L_c$  and the transparency. Position a mirror about 20 cm behind lens ( $L_1$ ) and perform the autocollimation procedure
3. Remove the point light source and mirror, turn the laser on, then place a viewing screen roughly one focal length beyond lens  $L_1$ . Adjust the viewing screen position until the diffraction pattern is at it's sharpest – the setup is now aligned and ready for use..
4. Remove the transparency and replace with each of the objects examined in Activity 2.1 - keep them at the same distance.

Which of the patterns observed in Activity 2.1 are closest to those produced by the lens, Fresnel or Fraunhofer?.....

Describe any notable differences between these patterns and those viewed in Activity 2.1  
 .....  
 .....

5. View and sketch the diffraction patterns from the following transparencies:



#4 Parallel lines (wide spacing)



#5 Parallel lines (medium spacing)



#6 Parallel lines (narrow spacing)

6. Sketch the diffraction pattern from Slide #5 (parallel lines) in diagonal orientations.



#5 Rotated clockwise (negative slope)

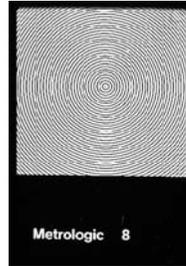


#5 Rotated counter-clockwise (positive slope)

7. Sketch the diffraction patterns from circular concentric lines.



#7 Concentric circles  
(wide spacing)

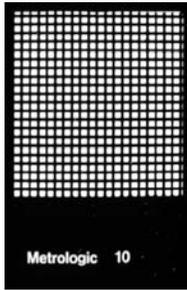


#8 Concentric circles  
(medium spacing)

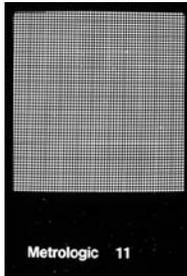


#9 Concentric circles  
(narrow spacing)

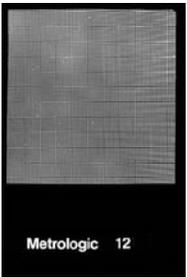
8. Sketch the diffraction patterns from grids of various spacing.



#10 Grid (wide spacing)



#11 Grid (medium spacing)

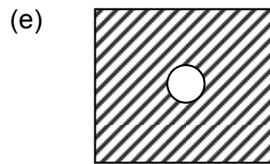
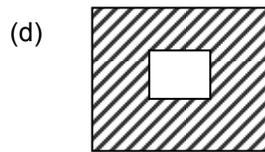


#12 Grid (narrow spacing)

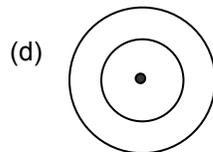
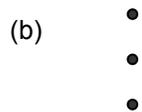
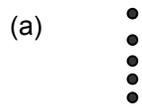
### Review problems

1. Sketch the set-up for viewing the Fourier transform of a transparency. Indicate any critical distances in units of lens focal length.

2. Sketch the diffraction patterns from the following:

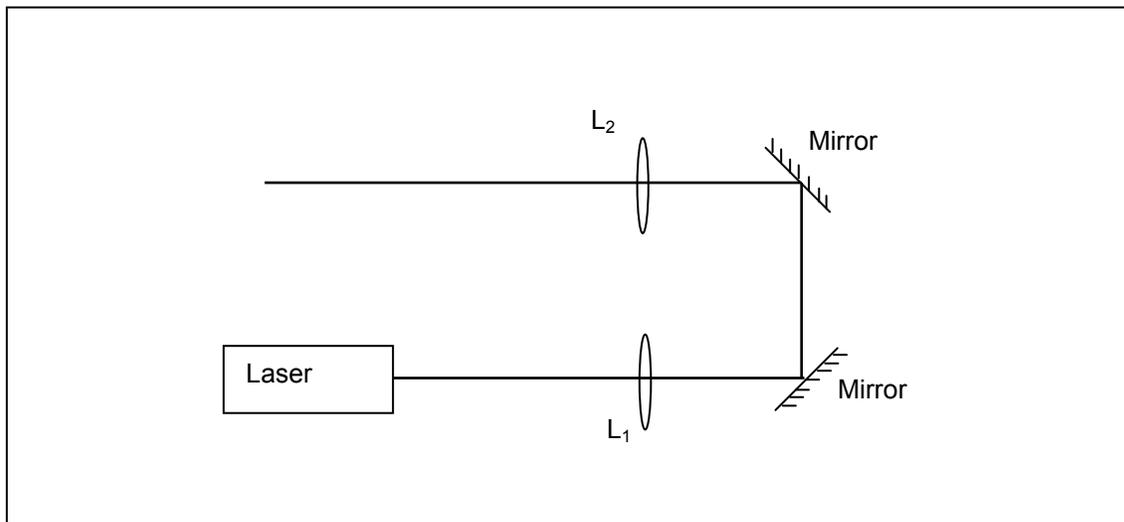


3. Sketch the transparencies that would generate these Fourier Transforms.



### SETTING UP A FOLDED OPTICAL BEAM PATH

The optical path length for the remaining activities is fairly long. Where this is impractical to establish you can use a folded optical beam path as shown below. A pair of front-surface mirrors is used to bend the beam back towards the laser to exploit available lab space. Place the front-surface mirrors between lenses  $L_1$  and  $L_2$ . Mirror position is not critical.



## ACTIVITY 3: DOUBLE DIFFRACTION – THE INVERSE FOURIER TRANSFORM

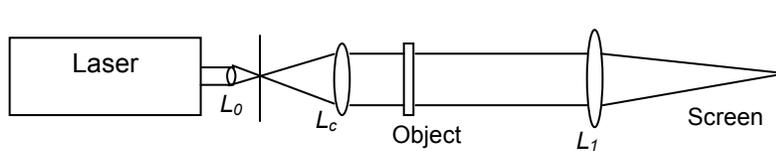
### Overview

In this activity you'll modify the previous setup to develop and view a second diffraction pattern and view an inverse Fourier Transformation.

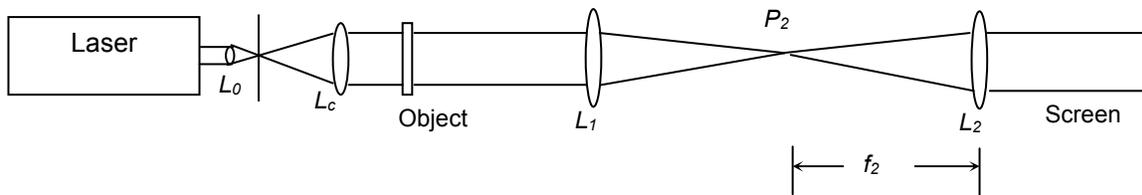
### ACTIVITY 3.1 VIEWING A DOUBLE DIFFRACTION

#### Background

In Activity 2.4 you generated the Fourier Transform of an object with a diffraction pattern at the image plane for lens  $L_1$  using the setup below:



In this activity you'll intercept the original diffraction pattern with another lens ( $L_2$ ) to produce a second diffraction pattern. The second lens is introduced into the original setup as shown below:



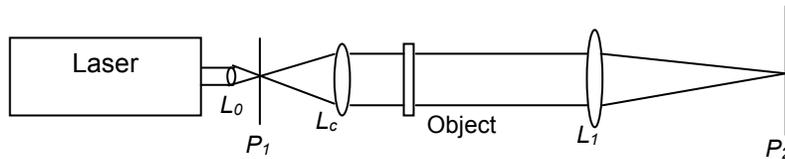
This “double diffraction” is an important result because it produces an inverse Fourier Transform, essentially recreating the original object image. This is an example of how the law of reciprocity for lens operation permits a reversal (or inversion) of the original Fourier Transformation. Once again no mathematics or signal processing is required, and you may start to realize the powerful implications of this process. The mechanism for this process is usually expressed mathematically in many textbooks - a brief overview is given in Appendix II.

To appreciate what's happening you might consider the operation of a digital camera or portable music player. Things you see or hear are converted to electrical signals which are not recognizable by you as having any relationship to the original (consider the Fraunhofer diffraction patterns you viewed in the previous activities as a similar occurrence). The electrical signals might be further converted to digital data, processed, then converted back to an “analog” form we can see and hear. The lenses in our setups perform the Fourier Transformation and its inverse without any electronics or data processing.

## Procedure

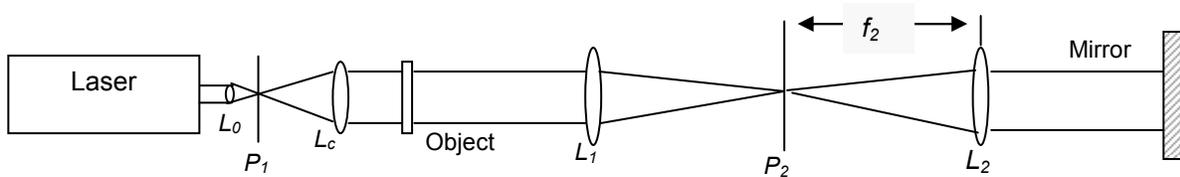
To properly position the newly introduced lens ( $L_2$ ) you'll need to setup another autocollimation alignment. The goal is to have both  $L_1$  and  $L_2$  positioned so their focal points converge at the same location ( $P_2$ ), which happens to be the F.T. plane. This can be done by using a pinhole as an object or target for the autocollimation at location  $P_2$ .

1. Make a small pinhole in a white index card with a needle or pin. It doesn't have to be precise since it won't be used as a spatial filter. You'll use it in the autocollimation process for  $L_2$ .
2. Remove the viewing screen used in the original setup from Activity 2.4 and replace it with pinhole  $P_2$  as shown below - (do not use an object for the initial alignment in steps 2 through 4):

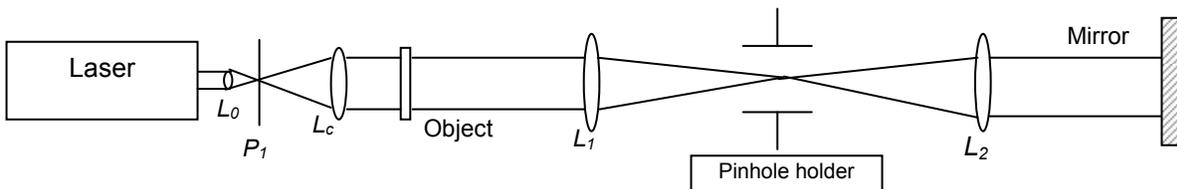


Carefully position the pinhole from  $L_1$  to bring the laser beam image into sharp focus on the index card. Then position the pinhole so that the focused laser beam passes through the opening.

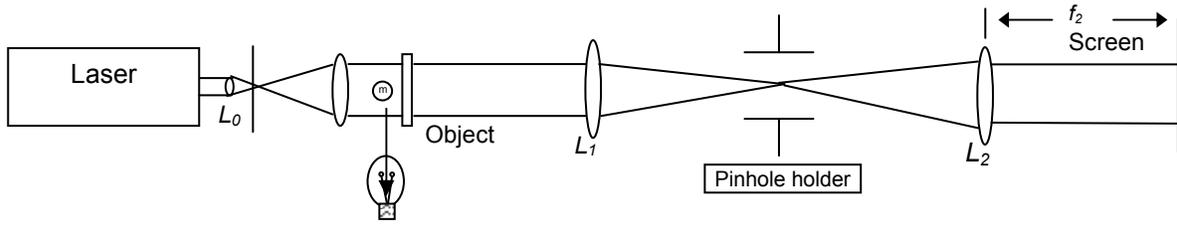
3. You now have a new target against which you can align lens  $L_2$ . Position lens  $L_2$  roughly one focal length away from the pinhole and precisely align it using the autocollimation technique from Activity 1.3 as shown below:



4. Carefully remove pinhole  $P_2$  without removing the pinhole holder. In later activities you'll place filters at this location (the F.T. plane) to perform image processing. Leaving the holder in place will eliminate the need for realignment – see below:



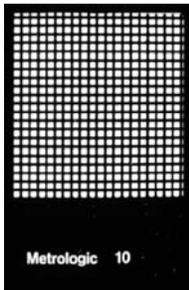
5. Turn off the laser and place a viewing screen at the image plane one focal length beyond the lens. Place a point source of white light (a small penlight or incandescent bulb) between the laser collimating lens and any transparency used as an object, then position the viewing screen until the image of the transparency is sharply focused on the screen.



Small or Penlight Bulb

Remove the point source, turn on the laser, and view the image. It should appear similar to the transparency but will be inverted. Distortions indicate that the setup doesn't work perfectly. In later activities we'll explain the limitations of this setup and what causes the distortions.

View and sketch the double diffraction images from the following transparencies. Note any distortions or issues with resolution.



#10 Grid (medium)

(Distortions if any)

---

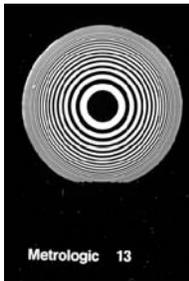
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#13 Concentric circles  
(variable widths)

---

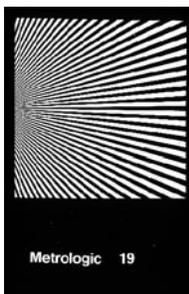
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#19 Fan pattern

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Describe any parts of the diffraction images which are distorted or do not resolve as well as others.

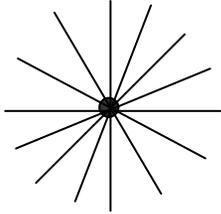
.....

.....

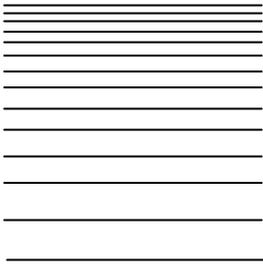
## Problems

Sketch the double diffraction image you would expect to see on the viewing screen from these object transparencies:

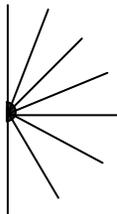
1



2



3



## ACTIVITY 4: ANALYSIS OF DIFFRACTION PATTERNS BY FILTERING

### Overview

In this activity you'll insert filters and masks at the F.T. plane in the optical setup from Activity 3. By isolating specific sections of a diffraction pattern you can determine their contribution to the total object image recreated by inverse Fourier Transformation. You'll discover in some cases that manipulating only a few elements in the diffraction pattern renders the recreated object image unrecognizable.

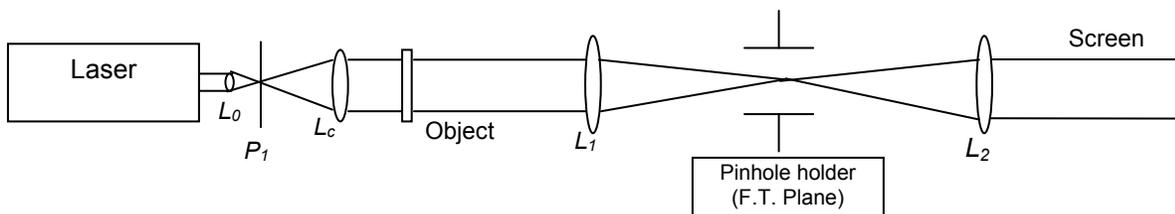
### ACTIVITY 4.1: FILTERING THE DIFFRACTION IMAGES OF A GRID.

### Background

In the next several exercises you will learn and exploit a very important aspect of the optical setup developed in the previous activities. The diffraction patterns at the F.T. Plane contain information about the object in what could be considered a disassembled format. Each dot or fringe in the pattern represents energy associated with a particular object characteristic. By intercepting and manipulating the pattern at the F.T. plane, you will affect the recreated image at the screen. This is basic image processing, and in later activities you'll apply these principles to more complex images. You'll gain an understanding of how powerful and important this concept is in everyday life.

### Procedure

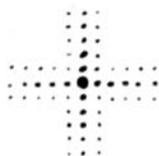
In Activity 3 you placed a pinhole in a holder at the F.T. plane in order to perform the autocollimation alignment for  $L_2$ . Now we'll insert various apertures and slits in the holder to isolate specific elements of the diffraction pattern and view the effects.



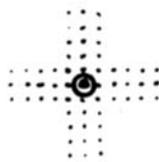
1. Use the setup from Activity 3 shown above and place transparency # 10 in the Object plane..
2. In the following exercises you'll use various masks to restrict which portion of the diffraction pattern will pass onto  $L_2$  before the inverse Fourier Transformation is performed. Use slide #3 (circular aperture), slide #15 (narrow slit), or slide # 16 (square apertures) in the pinhole holder to achieve the necessary restriction.

3. For exercises a through h place a mask in the pinholder to let only the indicated region of the diffraction pattern pass onto  $L_2$ , then sketch the double diffraction image and answer the questions.

a. No filter – Let the entire Fourier Transform pass.



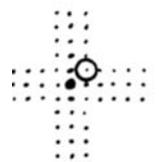
b. Let only the central dot pass.



Is the illumination uniform? .....

If not, describe.....

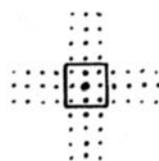
c. Let other dots pass one at a time.



Is the illumination from each dot uniform? .....

If not, describe.....

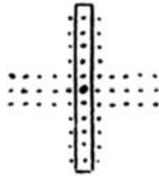
d. Let the central nine dots pass.



Does the corresponding double diffraction image look like a true representation of part or all of the original object? .....

If not, explain.....

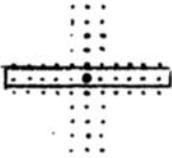
e. Let the central vertical row of dots pass.



Does the corresponding double diffraction image look like a true representation of part or all of the original object? .....

If not explain, .....

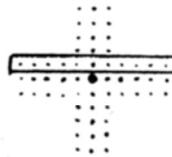
f. Let the central horizontal row of dots pass.



Does the corresponding double diffraction image look like a true representation of part or all of the original object? .....

If not, explain .....

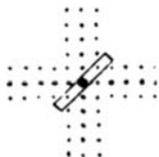
g. Let an off-center horizontal row of dots pass.



Does the corresponding double diffraction image look like a true representation of part or all of the original object? .....

If not, explain .....

h. Let a diagonal row of dots pass.



Does the corresponding double diffraction image look like a true representation of part or all of the original object? .....

If not, explain .....

**ACTIVITY 4.2: FILTERING THE DIFFRACTION IMAGES FROM CONCENTRIC CIRCLES.**

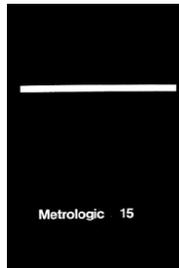
**Procedure**

Use the same set-up as Activity 4.1 but use transparency #7 (Concentric circles, wide) as the object and transparency #15 as the mask. Sketch the double diffraction images and answer the questions.

- a. No filter – Let the entire Fourier Transform pass.



- b. Horizontal slit.- Let only a horizontal cross section pass.



Does the corresponding double diffraction image look like a true representation of part or all of the original object? .....

If not, explain.....

.....

- c. Vertical slit. Allow a vertical cross section pass.



Does the corresponding double diffraction image look like a true representation of part or all of the original object? .....

If not, explain.....

.....

**Problems**

1. The previous exercises have given you experience with the Fourier Transform (diffraction patterns) formed by specific object features. You've also seen what type of images are produced by a particular single diffraction pattern when the inverse Fourier Transform is generated. You can now use this knowledge to predict a result in either direction from a particular object or diffraction pattern. Sketch the Fourier transform produced by the following letters:

Object

**A**

**E**

**K**

**L**

Single diffraction pattern  
(Fourier Transform)

Sketch the double diffraction image that would be formed for the following combinations of letters and filters:

(a)

Object

**A**

**E**

**K**

**L**

Filter



Double Diffraction Image

(b)

Object

**A**

**E**

**K**

**L**

Filter



Double Diffraction Image

2. Earlier in this activity you placed a slit diagonally across the diffraction pattern of a grid. The inverse Fourier Transform produced an image composed of diagonal lines which did not exist in the object. Explain how image features not present in the original object when it's unfiltered can appear in a filtered version.....

.....  
.....

*NOTE: There are similar manipulations that produce an image quite different from the original transparency. For example, by masking out the central dot of the Fourier Transform of a grating, the frequency of the grating will be doubled.*

## ACTIVITY 5: SPATIAL FREQUENCY

### Overview

In this activity you'll explore the relationship between the position of dots or fringes in a diffraction pattern to the information they convey about the object. The concept of spatial frequency is important in image processing and you'll see its impact by using a variable diameter iris or various apertures as filters. This will affect how some of the original object features are developed in the inverse Fourier Transform image

### ACTIVITY 5.1. FILTERING DIFFRACTION PATTERNS WITH APERTURES OR A VARIABLE – DIAMETER IRIS.

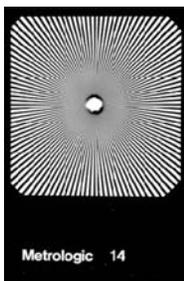
### Background

If you look at the F.T. plane the diffraction pattern generated by the object is spread in the X-Y coordinates around the focal point of  $L_1$ . In Activity 4 you performed exercises which gave you an indication that specific parts of the pattern were associated with certain object features. Blocking or passing particular groups of dots could have a dramatic effect on the reconstructed image. In the next 2 activities we'll concentrate on entire regions of the F.T. plane. We'll use a variable iris and apertures to initially restrict passage of the pattern to areas close to the center. Then you'll let increasingly larger areas of the pattern pass and compare results to an unobstructed condition.

### Procedure

1. Use the same setup from Activity 4 and place a variable iris at the F.T. plane. **Note: If your Physical Optics Set doesn't contain a variable iris then use the apertures in slides #17 and #18 to mimic the variable iris.**
2. Place each of the transparencies shown below in the object plane and view the double diffraction image on the screen. Gradually increase the diameter of the iris and view the image changes for each transparency. Describe the changes as the iris is enlarged (Hint: pay attention to the closely and widely spaced lines in the transparency)

Transparency

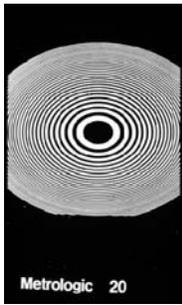


Describe the changes as the iris is enlarged

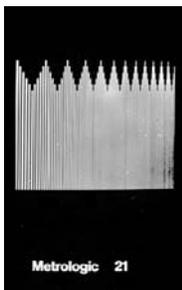
#14 Radiating lines

Transparency

Describe the changes as the iris is enlarged



#20 Oval lines (variable spacing)



#21 Parallel lines (variable spacing)

**Problems**

Is there a correlation between the location of the diffraction dots at the F.T. plane and which object features they represent? Are the dots linked to a particular position on the object or do they represent some other aspect? Review the changes you observed and give a detailed explanation of what object information is present in the central and outer diffraction dots at the F.T. plane.

.....

.....

.....

.....

.....



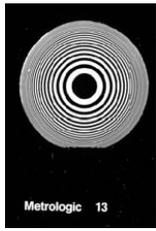
Where are the widely spaced lines in the object? .....

Where are the tightly spaced lines in the object? .....

Are widely spaced lines visible in the image when the filter aperture is:  
(1) small? ..... (2) large? .....

Are tightly spaced lines visible in the image when the filter aperture is:  
(1) small? ..... (2) large? .....

Object                      2 mm                      3 mm                      5 mm                      8 mm                      12 mm



#13 Concentric circles (variable spacing)

Where are the widely spaced lines in the object? .....

Where are the tightly spaced lines in the object? .....

Are widely spaced lines visible in the image when the filter aperture is:  
(1) small? ..... (2) large? .....

Are tightly spaced lines visible in the image when the filter aperture is:  
(1) small? ..... (2) large? .....

**Problem**

The transparencies have tightly and widely spaced lines which can be considered spatial information. Review your results and make a general statement regarding where spatial information from the object is encoded in the diffraction pattern at the F.T. plane. Make certain your explanation holds for all combinations of transparencies and apertures.

### ACTIVITY 5.3 SPATIAL FREQUENCY

#### Background

When we think of frequency it's usually in terms of how often something occurs in a given unit of time. Audible tones are related in cycles or vibrations per second, the sun rises and sets once per day, birthdays occur once per year, etc. But a broader definition of frequency is the number of times something occurs or changes (dependent variable) within a measurement unit (independent variable).

Spatial frequency relates to changes or variations vs. units of space or distance. How often a pattern repeats or varies in a specific distance is an example of spatial frequency. Simple real world examples of repeating physical patterns include regularly spaced telephone or power poles, fence posts, stairs, striped clothes etc. But it's important to note that all physical objects have some collection of spatial frequencies associated with their observable features, even if a pattern isn't immediately apparent. In other words, spatial frequency is not so much dependent on the shape of the feature (as in a line) but in how often it occurs within a fixed distance.

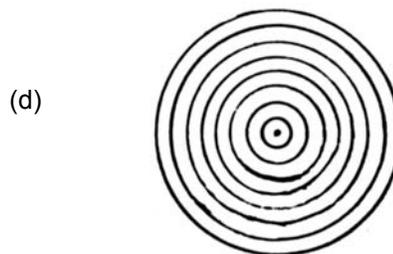
The pattern below has an easily determined spatial frequency:

Since the black-white pattern is repeated 6 times in 3cm, the frequency is 6 cycles per 3cm or 2 cycles/cm

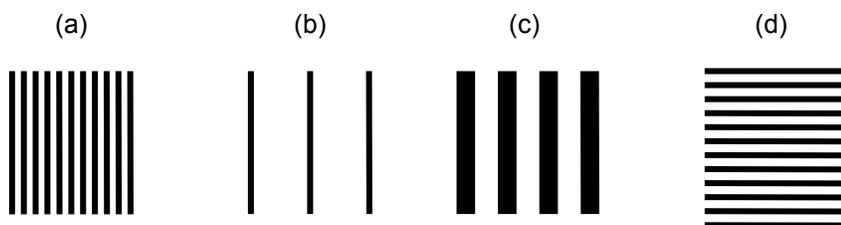


#### Problems

1. Find the spatial frequency of these patterns in cycles per centimeter.

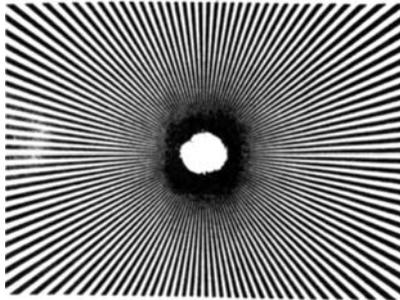


2. Which of the following has the highest spatial frequency? .....



3. The patterns below have variable spatial frequency. In which area of the patterns are the high and low spatial frequencies located?

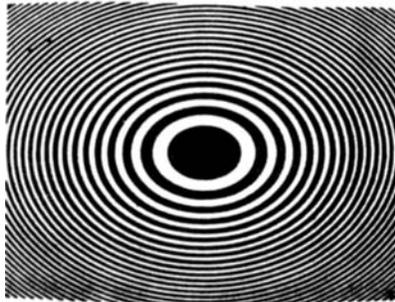
(a)



The high spatial frequencies are located at .....

The low spatial frequencies are located at.....

(b)



The high spatial frequencies are located at .....

The low spatial frequencies are located at.....

4. At the end of Activity 5.2 you were asked to make a general statement regarding where spatial information is encoded in the diffraction pattern at the F.T. plane. Your original statement should have been worded in terms of tightly spaced and widely spaced lines. Rephrase your answer in terms of spatial frequency:.....

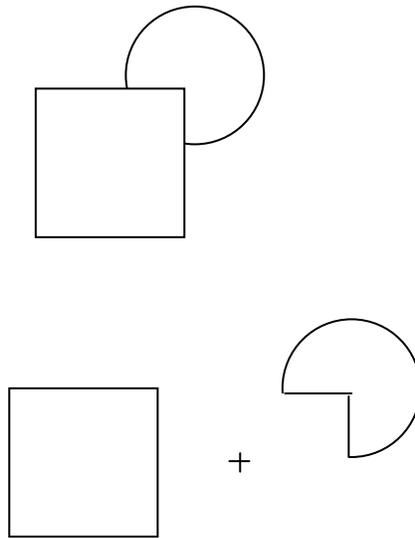
.....  
.....  
.....

**AN ALTERNATE THEORY**

There's an alternative way to understand the concepts and results of the exercises in Activity 5. The Ernst Abbe theory of image formation does not involve the use of Fourier Transforms or spatial filtering. It's more oriented towards lens behavior as applied to microscopes and other uses. You can find information on this theory on the Internet and in many textbooks on optics.

## A DIFFERENT PERSPECTIVE

It's important to note that the Fourier Transform is a representation of an object. The information it contains about the object is incomplete but can be extremely useful nonetheless. There are many examples in the world around us of representations, reproductions and symbols that provide information crucial to our daily existence. Photographs, audio recordings, radio and television broadcasts, cell phones, and even the text you're reading are all "virtual" - they are a projection of information from an "real" original object or source of information. Whether that information is everything you need to know about the original object depends on what you're trying to do, and in some cases, how you interpret what's presented.



The first drawing above may represent a square object in front of a circular object, but the second drawing shows it could also represent something different than your first interpretation. It's important to realize the limitations in any means of information transfer. Even when we see an object directly we're only *experiencing* one aspect – the visual image formed on our retinas and processed by our brains. To *know* more about the object may require touching, listening or smelling, among other possibilities.

It's been said we now live in an "Information Age" as if that's a recent evolution of the human experience. In reality human existence has always relied on information for survival and growth. Recent technology has provided new and exciting mechanisms for discovering and conveying information. Whether you're using Fourier Transformation, digital signal processing, ordinary speech, or any other mechanism keep a perspective on whether you're receiving what you need to know.

## ACTIVITY 6: IMAGE-PROCESSING APPLICATIONS

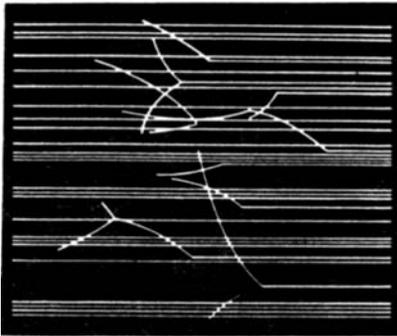
### Overview

In activity 4 you performed basic manipulation of the information content in an object image by intercepting and filtering the single diffraction pattern (Fourier Transform) at the F.T. plane.. In activity 5 you gained an understanding of where the information from certain object features is encoded at the F.T. plane. In activity 6 you'll perform image processing on objects closer to real world applications. The results will be more dramatic and help provide an appreciation of how useful Fourier Transformation is in everyday life.

### ACTIVITY 6.1. REMOVING UNWANTED DATA FROM A TRANSPARENCY USING OPTICAL FILTERING.

#### Background

Optical filtering of diffraction patterns can be used in practical applications. The cloud chamber photograph below has a variety of tracks associated with the incoming particles and the resulting interactions. Separating these from one another when viewing the photograph can be difficult. You can remove the incoming particle tracks from the recreated image by using some of the filtering techniques you learned in earlier activities. This allows easier viewing since only the interactions remain in the recreated image.



#### Procedure

Use the optical setup from Activity 5 with transparency # 22: (cloud chamber simulation) as the object and # 26 (bar without center) as the filter. Sketch the resulting image:



Slide #22



Filter

Resulting double diffraction image:

**Problems**

Describe what filter (slide #26) does to the recreated object image:.....  
.....  
What does the bar do?.....  
What does the central space do? .....

**ACTIVITY 6.2. CREATING A CONTINUOUS-TONE IMAGE FROM A HALFTONE DOT PATTERN.**

**Background**

A halftone photograph is a representation of an image that originally contained a continuous grey scale from the lightest to darkest areas. If you wonder why halftone images were developed consider the early printing presses. The technology was only capable of laying down black ink on paper to form clearly defined letters or line art. In a certain sense it was an early “digital” format because the ink was either fully applied (digital high), or not there at all (digital low) – there was no intermediate condition or shade of grey

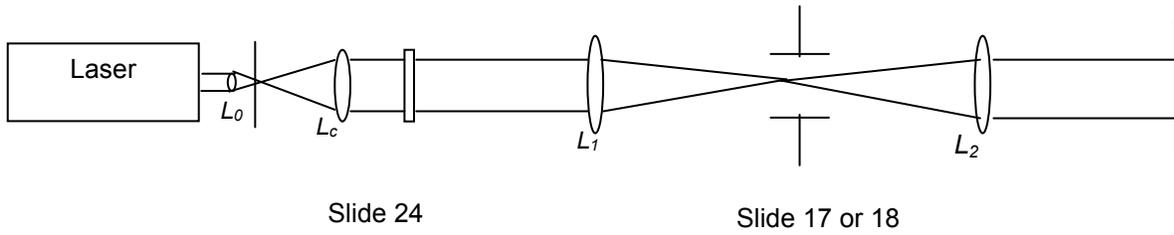
Halftone techniques were developed to overcome this limitation so that real world images or photographs could be reproduced with early printing processes and still appear “natural”. Early photographic versions of this technique was conceptually simple - a mask or screen with regularly spaced apertures was placed between the image and camera. This generated a pattern of dots in the form of the image at the camera film or image plane. The size of the dots corresponded to the light/dark intensity of the image at the particular screen location. Darker areas formed smaller dots and light areas formed larger dots. The film was developed into a negative that was used to transfer the dot pattern to a photosensitive printing surface. In one variation the surface is etched, producing a recreated image composed of dots. Ink is applied to the drum which was transferred to paper with a press. If you look at newspaper photographs with a magnifying glass you’ll see a real world example of this concept. From a distance the halftone photographs in a newspaper appear continuous. Below is an example of a halftone photograph:



Aspects of the preceding description should remind you of concepts presented in earlier activities. Halftone image processing is a practical example of the filtering and image recreation exercises you

performed. In this activity you'll use these techniques to recreate the original continuous tone image represented by the halftone photograph.

**Procedure**



In the setup above use slide # 24 (halftone photograph) as the object and a variable iris or slides # 17 & 18 (circular apertures) as the filter.

Vary the iris diameter or aperture sizes and describe changes in the image as larger diameters are used:  
 .....

**Problem**

View the recreated image of halftone slide #24 using slide #15 at various angles as the filter. For each angle predict what you think the image should look like, then view and sketch the actual recreated image.

	<u>Halftone</u>	<u>Filter</u>	<u>Predicted Image</u>	<u>Observed Image</u>
(a)		None		
(b)				
(c)				
(d)				

## ACTIVITY 6.3 RECOVERING MULTIPLE IMAGES FROM A SINGLE STORAGE TRANSPARENCY.

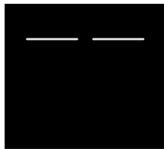
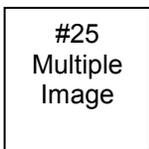
### Background

Occasionally there's a need to store more than one image on a viewing surface. One example is the holographic images placed on credit cards or driver's licenses. For security purposes it might be desirable for the second image to be hidden or not immediately apparent to the viewer. Then the image can be retrieved using special equipment for security verification purposes.

One way to store multiple images on a photograph is to filter each image projection using a unique set of lines at different angular orientations. You can expose the film using horizontal lines for one image, and vertical lines for a second image. Then by using a filter with horizontal or vertical lines we can independently view each image on the photograph.

### Procedure

Using the previous setup choose slide #25 (multiple image photograph) as the object and slide #23 (slit without center) as the filter (see below).



Hold the filter in a vertical position. Describe the image which appears on the viewing screen.

.....

Hold the filter in a horizontal position. Describe the image which appears on the viewing screen.

.....

### Closing thoughts

The intent of these activities was to provide you with a practical perspective on Fourier Transformation. In addition you'll hopefully have gained a more intuitive understanding of the transformation process. This should help you understand or at least relate to information and signal processing concepts you might encounter in many real world applications. An example would be advanced image processing software applications available to manipulate digital camera images, like Adobe Photoshop. Remember that regardless of whether the manipulations are done using computer algorithms on digital data, or with analog techniques, the fundamental principles are the same.

## SELECTED ANSWERS TO ACTIVITIES AND PROBLEMS

### ACTIVITY

- 2.1 Sketch the diffraction patterns from some common objects: Fresnel patterns look like the original object but have lines around the outline. At close distances the interference patterns from all the object features haven't fully developed. Fraunhofer patterns don't resemble the original object image, but are the fully developed interference patterns from the object features projected at far distances. Typical sketches might look as follows:



Tip of needle  
(10cm)



Foil with holes  
(10cm)



Foil with holes  
(10 meters)

- 2.2 As the gap between the razor blades decreases, the spacing between the bright diffraction fringes increases. You can find the mathematical relationship for the effect in a physics text or Industrial Fiber Optics "*Experiments using a Helium Neon Laser*".
- 2.3 The sketch for the transparency with eight holes in a circle should resemble that of the foil with eight holes in Activity 2.1.
- 2.3 Problem: Fresnel diffraction patterns resemble the shape of the object. Fraunhofer patterns have lines and dots which don't look like the object. Fresnel patterns are complicated and blurry. Fraunhofer patterns are sharp and distinct.
- 2.4 The patterns in this activity are closest to the Fraunhofer patterns viewed in Activity 2.1. The lens provides an alternate method for viewing Fraunhofer patterns without using large physical distances. There are no significant differences between the patterns viewed in this activity and in 2.1. Lenses with a longer focal length will cause the pattern to spread out further.
- 2.4 The narrower the spacing between the lines, the greater the distance between the diffraction dots. The diffraction pattern sketches for the transparencies should resemble the the following photographs:



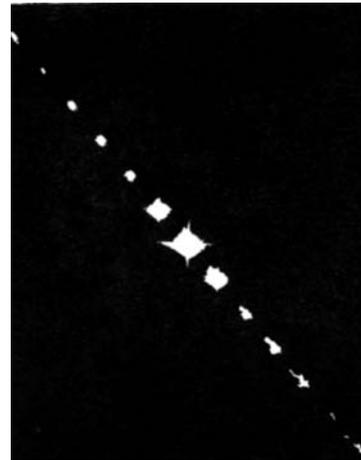
Parallel lines  
(medium spacing)



Photograph of  
diffraction pattern

### ACTIVITY

2.4 (6) Rotated orientations of parallel lines: The diffraction pattern will have a line of dots oriented opposite the original's rotation angle. If the original lines slope upward, the diffraction dots slope downward.



2.4 (7) Circular concentric lines: The narrower the concentric circle spacing on the transparency, the wider the ring spacing in the diffraction pattern.



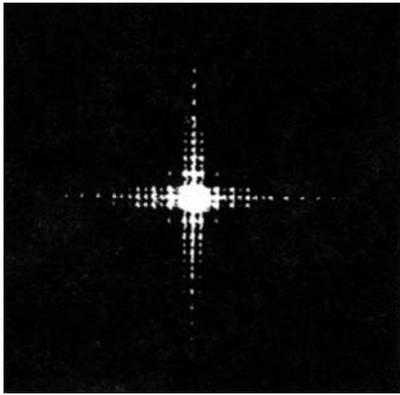
From Slide #7 (wide spacing)



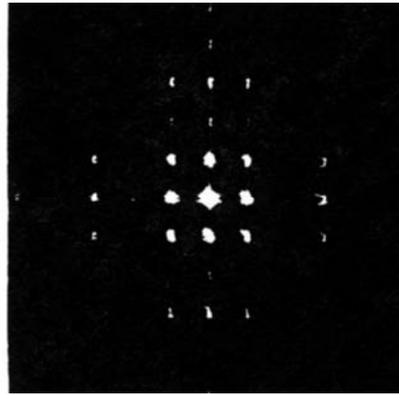
From Slide #9 (narrow spacing)

ACTIVITY

2.4 (8) A grid with wide spacing produces a diffraction pattern with tightly spaced dots. A grid with narrow spacing produces a diffraction pattern with widely spaced dots.



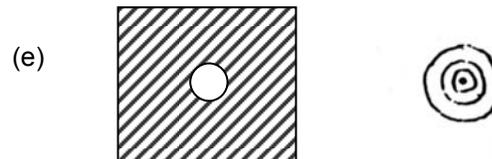
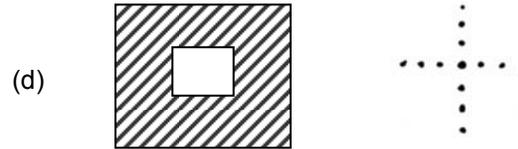
From Slide #10 (wide spacing)



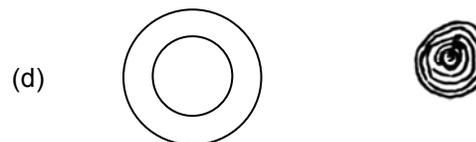
From Slide #12 (narrow spacing)

2.4 Problems

Objects and their associated diffraction patterns:



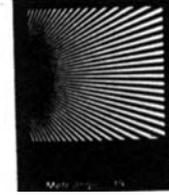
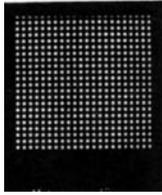
20 Diffraction patterns and their associated objects:



ACTIVITY

3.1 View and sketch diffraction images:

Object transparencies:



Corresponding images:

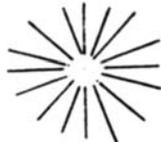


3.1 The diffraction patterns resemble the transparencies, but are reversed. The high frequency (closely spaced) parts of the slides don't resolve as well as the low frequency portions.

3.1 Problems:



Object



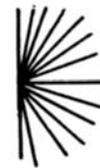
Image



Object



Image



Object



Image

4.1 Filtering the diffraction image from a grid:

(a) With no filter the entire Fourier transform passes and reappears as a grid shown in 'A' below.

(b) Allowing only the central dot to pass gives a relatively uniform non-distinct image as in "B".

(c) Passing other single dots gives results similar to "B".

(d) Passing the central nine dots yields a grid

(e) Passing a vertical row of yields horizontal lines as in "C".

(f) Allowing only horizontal dots to pass yields vertical lines.

(g) Passing an off-center horizontal row of dots yields vertical lines.

(h) Passing a diagonal row of dots yields diagonal lines sloping in the other direction. Note: There were no diagonal lines in the original image!

A



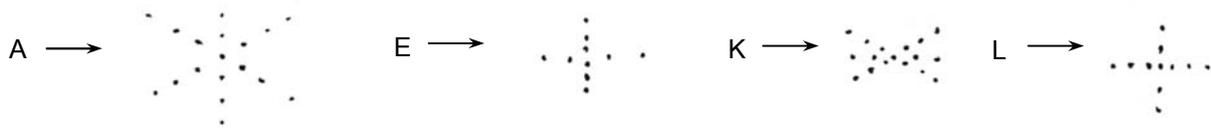
B



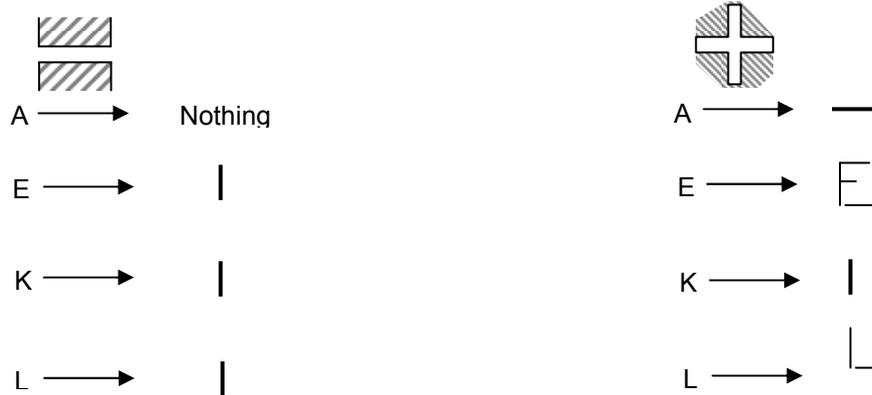
ACTIVITY

4.2 Problems:

4.2 (1) Predicted single diffraction images from letter objects:

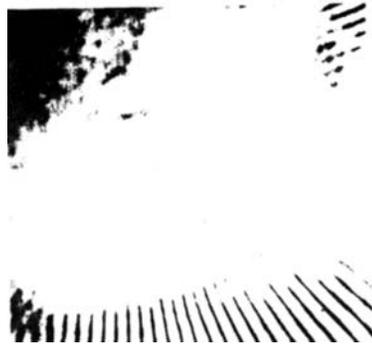


4.2 (1) Predicted double diffraction images from letter objects and filters:

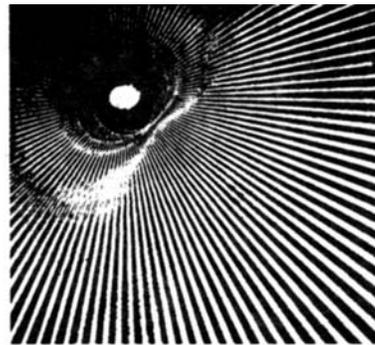


4.2 (2) By isolating certain dots, destructive interference from all the original diffraction dots formed by the object is no longer present. Object features not present in the original may appear.

5.1 At narrow iris diameters, widely spaced object parts are transmitted but finely spaced parts are blurred. As the iris is progressively opened, finer details become visible in the double diffraction image.



Small Iris



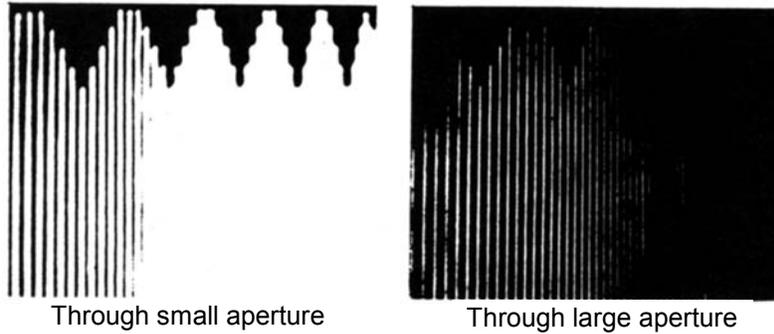
Large Iris

5.1 Problems: General statement on spatial information: The center of the single diffraction pattern contains information from the widely spaced lines or features of the object, and the outer part corresponds to the closely spaced lines

### ACTIVITY

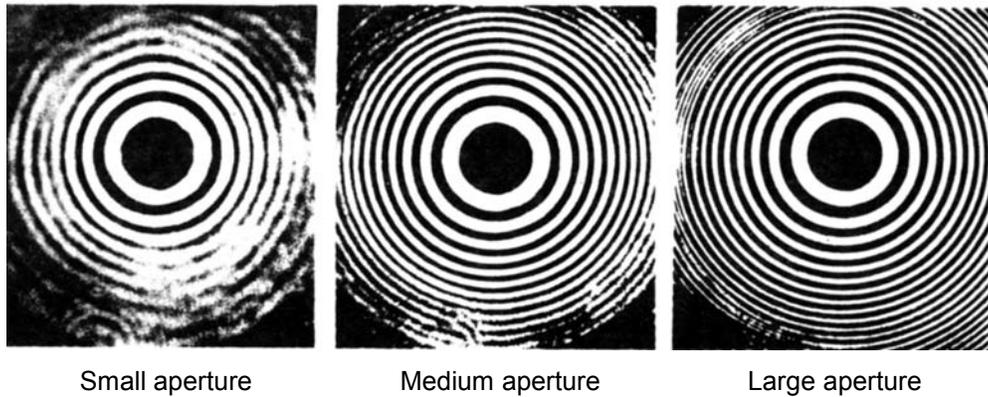
5.2 Fan pattern # 19 The widely spaced lines are visible through both large and small apertures. The closely spaced lines are visible only through the large apertures.

5.2 Parallel lines # 21, Photos below:



5.2 Concentric Circles # 13 (variable spacing)

The closely spaced lines are visible only through the larger apertures. Photos below.



5.3 Problems:

- 1 (a) 1 cycle/cm (b) 1 cycle/cm  
(c) 4 cycles/cm (d) 4 cycles/cm
- 2 "a" has the highest spatial frequency.
- 3 (a) High frequency at center; low, at edge.  
(b) High frequency at edge; low, at center.
- (4) The outer portions of the Fourier diffraction pattern are caused by closely spaced lines which correspond to high spatial frequencies. In general, low spatial frequencies are encoded in the central area of a diffraction pattern, and high spatial frequencies are carried in the outer portion.

## ACTIVITY

### 6.1 Procedure:

The recreated image sketch from the cloud chamber investigation slide has no horizontal lines and shows only the curved ones associated with the particle interactions.

#### Problems:

The bar in the filter eliminates the central column of dots causing the horizontal lines to disappear.

The central space allows energy common to the horizontal and curved lines to pass through, but since the information associated with the horizontal lines has been stripped only the curved ones appear.

### 6.2 Procedure

As the diameter of the aperture increases, the diffraction pattern becomes more detailed. The smaller the diameter of the aperture, the more the image resembles a continuous tone photograph.

#### Problem

The Fourier transform will look like a checkerboard pattern of dots.

- a. With no filter, a halftone image will appear.
- b. With a horizontal filter, the image will have vertical lines through the image.
- c. With a vertical filter, there will be horizontal lines.
- d. With a diagonal filter, there will be diagonal lines.

### 6.3 Problem:

The slide contains a male and female face. With no filter in place both images appear. One face appears when the filter is held either horizontally or vertically.

## APPENDIX I. CREATING OBJECT TRANSPARENCIES

**Note:** The transparencies provided with the Physical Optics Set were originally created using photographic film techniques. Modern technology provides new mechanisms for this process including but not limited to digital cameras, laser printers, and a variety of image processing and graphic design software applications. As is the case for many topics this can be researched on the Internet or the information provided by your instructor.

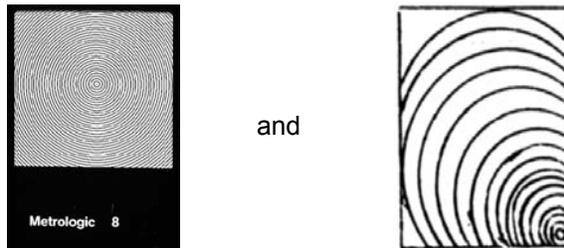
### How the transparencies were created:

Many of the transparencies discussed in this book were photographed from prints of Moire patterns.

The transparencies provided with the Physical Optics Set were photographed on Kodalith film. This film was selected because the black images are very dense, the white is clear, and the lines are sharp.

The transparencies used in the original project were photographed using Kodak Panatomic-X film, which has a slightly less dense “black”. A 35 mm single lens reflex camera with an internal exposure meter was used. The camera meter was used to ascertain a target exposure. Several photographs were taken. The target exposure bracketed exposures 1 and 2 F-stops darker and brighter. A good exposure is one with dense black, clear white, and sharp lines (no bleeding).

By varying the magnification or placement of a pattern, you may in fact discover other fields for experimentation. For instance, the patterns may lead to the investigation of the translation of an object and its effect on the Fourier Transform.



### PHOTOGRAPHING GRIDS

The grids of perpendicular lines supplied in the physical optics set were made by double exposure. The image was parallel lines. The film was exposed. The parallel lines were rotated 90°. The film was then exposed again. Bracketing above and below the target exposure reading is recommended to achieve the optimum exposure. All of the grids supplied in the Physical Optics Lab were made from the same set of parallel lines. The camera was simply moved closer to or farther away from the image to change the magnification.

### PHOTOGRAPHY OF PATTERNS AT THE IMAGE PLANE

It may be useful for classroom discussion to photograph the Fourier transform and/or the image plane. To accomplish this, use a 35 mm single lens reflex camera with the lens removed. Kodak Panatomic-X film is recommended. This film has a very fine grain that captures details of a good density, which makes

the background appear quite black. Use the camera's exposure meter to determine a target exposure setting. Again, bracket exposures over and under the target. The use of a cable release to prevent vibrations is recommended.

## **MASKS**

Masks or filters provided in the physical optics set have been photographed on Kodalith film. Home-made masks can be made with opaque Scotch photographic tape.

## **PHOTOGRAPHY OF MULTIPLE IMAGES**

To make an object transparency that stores two images on the same piece of film, try the following method:

Place a grating such as Slide # 4 (parallel line, wide spacing) over a continuous-tone photographic transparency. Copy (using a 1:1 ratio) onto a piece of continuous-tone film. Replace the photographic transparency with a second transparency. Rotate the grating 90°, and expose on the same piece of film. the exposure of the film may be determined by trial and error rather than callibrating the slope of the developing curve. Underexposure usually works best.

## **PHOTOGRAPHY OF HALFTONES**

Try various halftone images; copy newspaper photographs at different magnifications. Panatomic-X is recommended.

## ADDENDUM

### HELPFUL HINTS

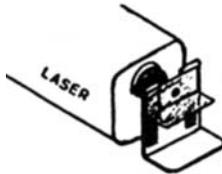
#### Mounting the lens on the laser:

If your laser doesn't have a threaded optics mount to accept the +15 mm focal length lens, it can be attached to the laser housing near the beam aperture with strips of magnetic tape.



#### Pinhole:

Attach the pinhole to a double post mount without an additional base attachment so that it can be moved close enough to the laser.



#### Other optical components:

Attach all lenses, slides, filters, etc to a double post mount with a base attachment for stability.



#### Lenses and Slides:

Lenses require a double thickness of magnetic tape to secure them to the double post mount.

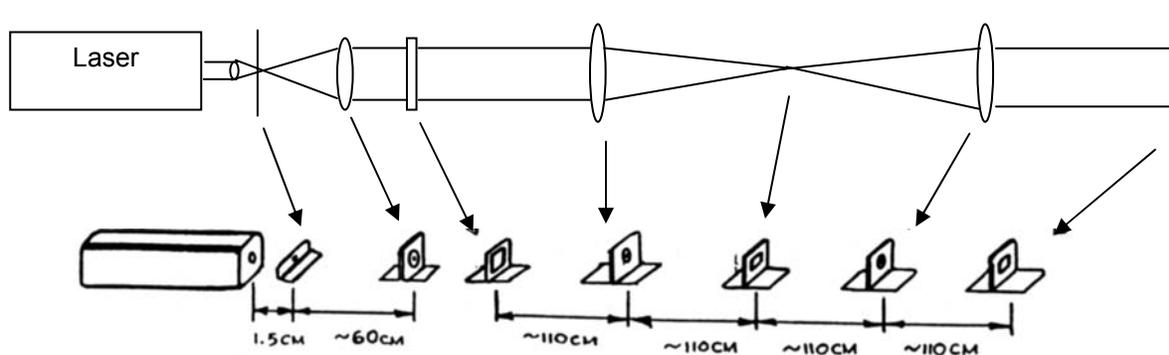


While slides require only a single thickness of magnetic tape.



(Cut magnetic tape with a scissor or score with a razor blade and break. Remove paper backing and press adhesive side against lens or slide.)

Below is a pictorial view of the double diffraction setup:

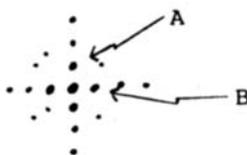


### Viewing patterns:

For viewing patterns without a magnifier a white index card can be used at the appropriate position. To look at the pattern with a magnifier the frosted mylar viewing screen should be used.

### Activity 6.3

If the image is bright compared to the surroundings, each of the single portraits in the multiple image slide can be viewed by filtering out all but one bright spot. The diffraction pattern (Fourier Transform) of the multiple image slide is:



Allowing only spot "A" to pass gives one portrait, while allowing only spot "B" gives the other portrait. (Slide #3 can be used as the filter).

### For classroom demonstrations:

All of the experiments can be used as classroom demonstrations and made visible to students through the use of a video camera (or camcorder) and monitor. It's usually best if the image can be developed right at the sensor plane of the video camera. But that can be impractical because lenses are often integrated in digital camcorders and cameras. An alternative is to focus the digital camcorder, camera, etc directly on ground glass, thin paper, or diffuse mylar placed at the image plane of the optical setup. You should consider experimenting with the vast array of digital video and still camera devices available now since their cost has dropped dramatically in recent years. A webcam might be a convenient way to feed the images from the activities into a network of computers accessible by students.

## APPENDIX II. MATHEMATICS OF THE FOURIER TRANSFORM

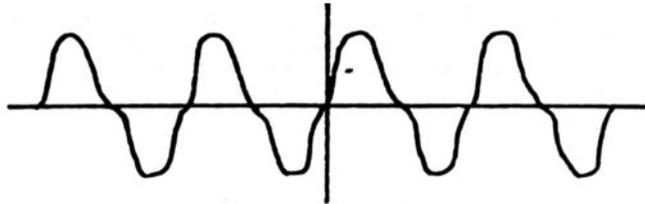
**Note:** For continuity, Arther Eisenkraft's original mathematical description of Fourier Transformation is included in this manual. Additional materials on this subject should be provided by your instructor or can be researched on the Internet.

### The Fourier Transform

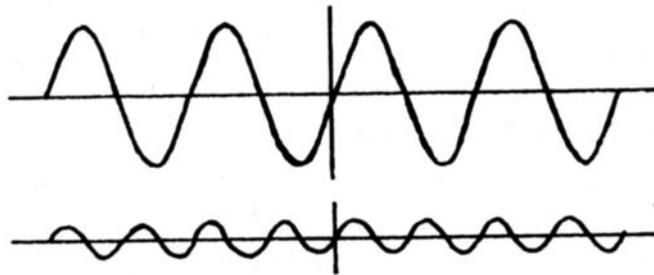
The following is a brief qualitative description of Fourier analysis. Other treatments, both qualitative and quantitative are available in numerous texts in physics, mathematics and engineering.

The mathematical theorem of J.B. Fourier can be basically stated: A periodic function can be represented by the sum of sinusoidal functions. To be more specific, the function  $f(x)$  with period  $P$ ,

below, can be represented by the sum of sinusoidal functions with periods  $\frac{P}{2}, \frac{P}{3}, \frac{P}{4}$ , etc.



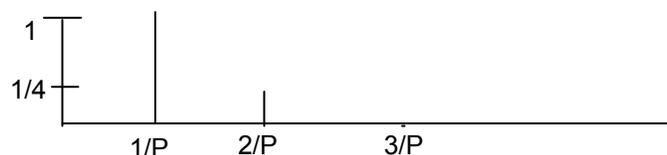
In fact, as you can see, the addition of the two sine waves below yield the given function  $f(x)$ .



You will notice that the amplitude of the two sine waves is not identical. In fact, the wave of twice the frequency (half the period) of the function  $f(x)$  has only one-fourth the amplitude.

This can be represented graphically in a "spectrum" chart of the function  $f(x)$  by using the abscissa to represent the frequencies of the components and the ordinates as the amplitudes of the components.

In case of our function  $f(x)$  of frequency  $\frac{1}{P}$  the "spectrum" chart would be:



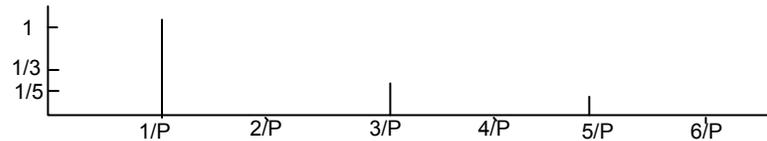
## Square Wave

A square wave, which varies between +1 and -1 and has period  $P$ , can be represented by the Fourier series:

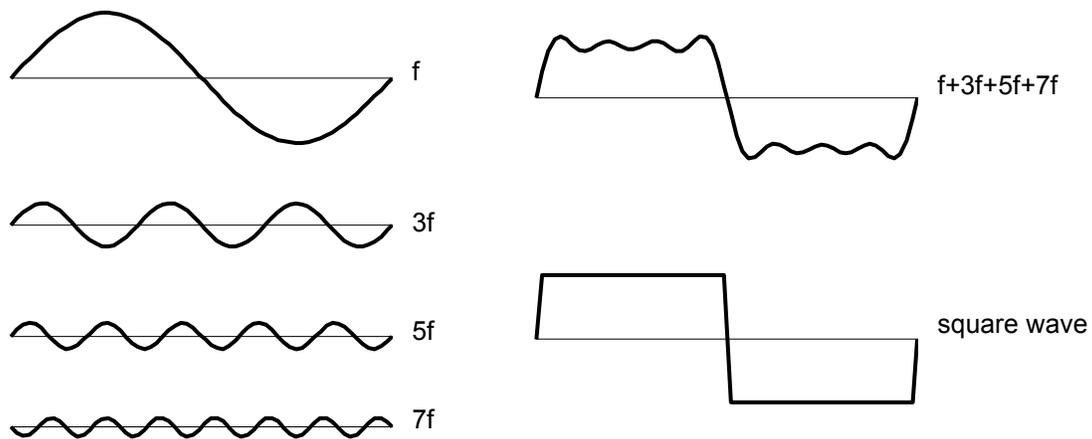
$$f(x) = \frac{4}{\pi} \left( \sin \frac{2\pi x}{P} + \frac{1}{3} \sin 3 \frac{2\pi x}{P} + \frac{1}{5} \sin 5 \frac{2\pi x}{P} + \dots \right)$$

Although there are an infinite number of components, the addition of the first four can be shown to yield the basic square wave form. The addition of higher and higher terms leads to a better representation.

The spectrum of the square wave is:



A graphical approximation of the addition of the first four terms is below:



## Fourier Integral

M. Francon in "Diffraction, Coherence in Optics" states; the spectrum of the function  $G(x)$  can only be represented by a set of discrete signals of periodic functions, but the concept of a spectrum can be generalized for the case of non-periodic functions. Any function which is everywhere finite and integrable can be represented by the sum of an infinite number of sinusoidal components.<sup>6</sup>

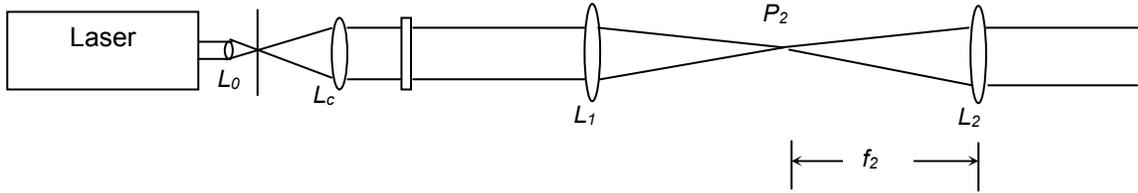
The relation between the function  $G(x)$  and its spectrum is the Fourier transformation. For example, consider the sketch of a slit function and its Fourier transform:



<sup>6</sup> New York: Pergamon Press, 1966, p.28.

**Mathematical representation of Double Diffraction:**

Although the mathematics will not be used in these exercises, a brief introduction to two types of notations for expressing double diffraction may help in reading and understanding other literature.



Lens  $L_1$  computes and displays the Fourier Transform of the object (the diffraction pattern).  
 Lens  $L_2$  takes the inverse Fourier transform of the function in its object plane.  
 This may be symbolically expressed:

$$F.T^{-1}[F.T.(f_1(x,y))] = f_1(x,y)$$

The Fourier Transform ( $F.T.$ ) is taken of function  $f_1(x,y)$ . The inverse Fourier Transform ( $F.T.^{-1}$ ) is then taken of  $F.T.(f_1(x,y))$ . this will yield the original function  $f_1(x,y)$ .

In other literature, the same analysis may be symbolically expressed:

$$f(x,y) \xrightarrow{F.T.} F(u,v) \xrightarrow{F.T.^{-1}} f(x,y)$$

$f(x,y)$  gives Fourier Transform  $F(u,v)$ .  $F(u,v)$  gives by inverse Fourier Transform  $f(x,y)$ .

### APPENDIX III. ADDITIONAL RESEARCH

The following are the original references provided by Arthur Eisenkraft in previous versions of this manual. They are reprinted here for the sake of continuity, but are generally at an advanced level that may not be appropriate for students using this manual. IFO recommends that students follow the advice of their instructors for more current materials on this topic. The Internet is also an excellent resource for additional research, but since the content is constantly changing any website information we provide would become quickly dated. We recommend beginning any additional research on this topic at websites associated with University or Government laboratories.

Original textbook recommendations:

Hecht, E., Zajec, A., "Optics".

Meter-Arendt, J.R., Introduction to Classical and Modern Optics.

Although these texts are involved, they cover the topics in this project extensively.

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## PARTS LIST: INDUSTRIAL FIBER OPTICS PHYSICAL OPTICS SET # 45-688

NUMBER	QTY	DESCRIPTION
45-629	1	Lens ( $L_o$ ): +15mm focal length mounted in a $\frac{3}{4}$ "-32 TP1 threaded cell. This lens may be screwed into the threaded optics mount on Industrial Fiber Optics ML-8xx lasers. The lens serves both to focus the beam through a pinhole to form a spatial filter, and to function as the eyepiece lens of a Galilean-type collimating telescope.
45-664	1	Lens ( $L_c$ ): +1.7 diopter spherical eyeglass lens mounted on a square plate painted gray. The lens serves to form a collimated beam and functions as the objective lens of a Galilean-type collimating telescope.
45-665	1	Set of 2 Lenses ( $L_1$ and $L_2$ ): +0.9 diopter spherical eyeglass mounted on a square plate painted black. These long focal length lenses are used to create the diffraction patterns and images.
45-647	1	Set of 2 Mirrors, 70 mm square. Front surface aluminized and overcoated.
45-649A	8	U-shaped Double Post Mount (to hold the slides, pinholes, lenses and viewing screens). The components will be attached magnetically to the steel mounts.
45-650A	8	L-shaped Mount used as Base Attachment (to be used with each mount above for vertical stability). Each base attachment contains a magnetic strip which holds it firmly against the post mount.
00-00160	4	Footing strips with adhesive backing. Each strip is 250 mm long and should be cut with scissors and attached to the feet of the post mounts and base attachment. The footing strips help prevent movement when the mounts are used on smooth surfaces.
45-660	5	Magnetic strips with adhesive backing. Each strip is 150 mm long. The strips should be cut with scissors and attached to the slide, pinhole, lens, and mirror holders.
45-671	1	Set of 2 Viewing Screens. Frosted Mylar viewing surface in 2x2" slide mount.
45-673	1	Set of 26 transparencies. (Individual transparencies are not available separately). Each transparency is mounted in a 2x2" slide holder. <ol style="list-style-type: none"> <li>1. Eight holes in a 5 mm circle</li> <li>2. Square aperture (2 mm)</li> <li>3. Circular aperture (2 mm)</li> <li>4. Parallel lines (wide)</li> <li>5. Parallel lines (medium)</li> <li>6. parallel lines (narrow)</li> <li>7. Concentric circles (wide)</li> <li>8. Concentric circles (medium)</li> <li>9. Concentric circles (narrow)</li> <li>10. Grid (wide)</li> <li>11. Grid (medium)</li> <li>12. Grid (narrow)</li> <li>13. Concentric circles (variable spacing)</li> <li>14. Radiating lines</li> <li>15. Slit</li> <li>16. Square apertures</li> <li>17. Circular apertures (2, 12 mm dia.)</li> <li>18. Circular apertures (3, 5, 8 mm dia.)</li> </ol>

19. Fan pattern
20. Oval lines (variable spacing)
21. Parallel lines (variable spacing)
22. Cloud chamber simulation
23. Slit without center
24. Halftone photograph
25. Multiple image photograph
26. Bar without center

45-674	1	50 micrometer hole mounted in magnetic tape.
45-788	1	The Physical Optics Manual

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